

Energy Conversion

2nd year electrical

Chapter 3: Transformers

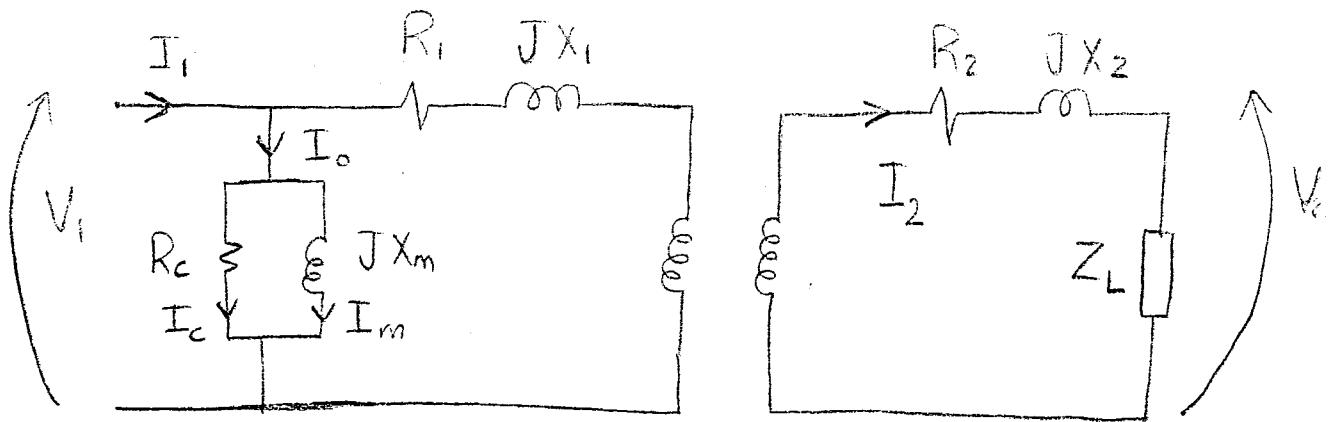
Part (2)

Paper:

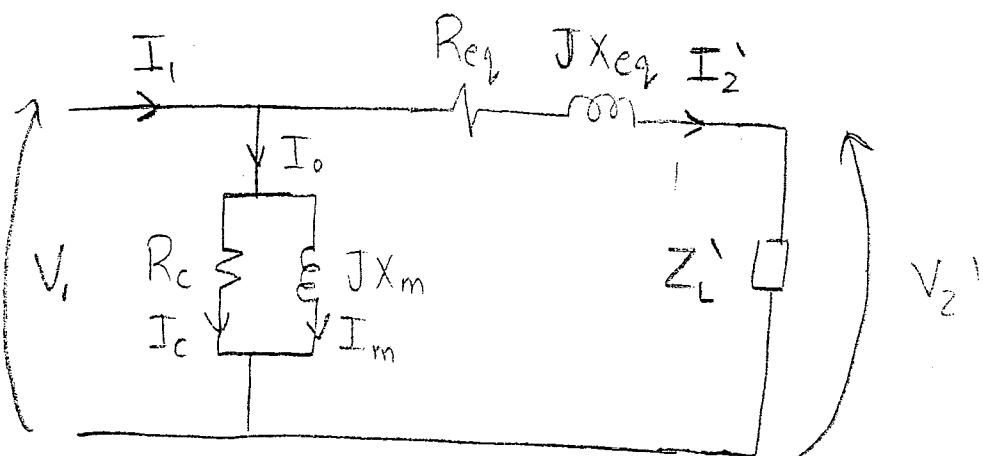
Transformers - Continued

- ① Approximate equivalent circuit
- ② Voltage regulation + phasor diagram
- ③ Efficiency (η)
- ④ Transformer tests
 - open circuit test
 - short circuit test
- ⑤ Separation of iron losses
 - Hysteresis
 - Pddy

① Approximate equivalent circuit



→ Referring to primary side



$$* R_{eq} = R_1 + R_2'$$

$$R_{eq} = R_1 + a^2 R_2$$

$$* X_{eq} = X_1 + X_2'$$

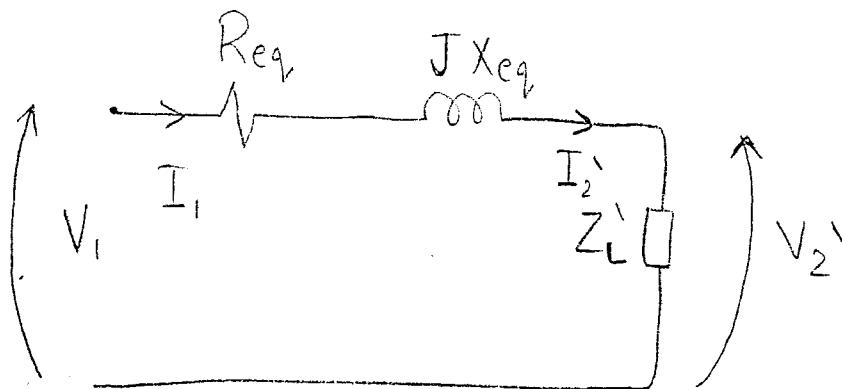
$$X_{eq} = X_1 + a^2 X_2$$

①

② Transformer Voltage regulation (VR)

Definition: It's a measure of the change of Voltage From no load to any loading Condition. ($VR \leq 5\%$)

→ Consider the equivalent circuit of the transformer as follows:



$$\text{Voltage regulation} = \frac{V_1 - V_2'}{V_1}$$

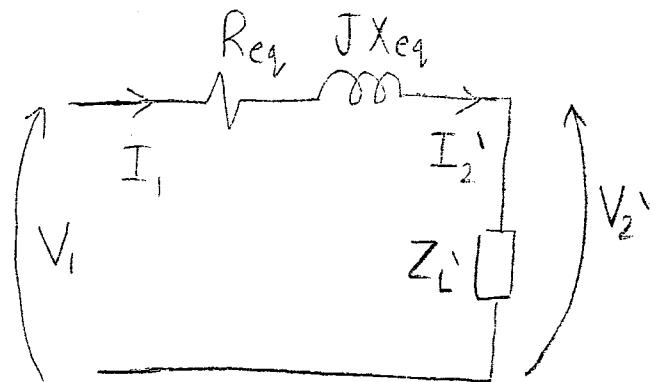
$$\rightarrow \underline{\text{at no-load}} \rightarrow I_2' = 0 \rightarrow V_1 = V_2'_{\text{no-load}}$$

$$\therefore \text{Voltage regulation} = \frac{V_1 - V_2'}{V_1} = \frac{V_2'_{\text{no-load}} - V_2'}{V_2'_{\text{no-load}}}$$

(2)

Voltage regulation for lagging p.f. load

$$VR = \frac{V_1 - V_2'}{V_1}$$

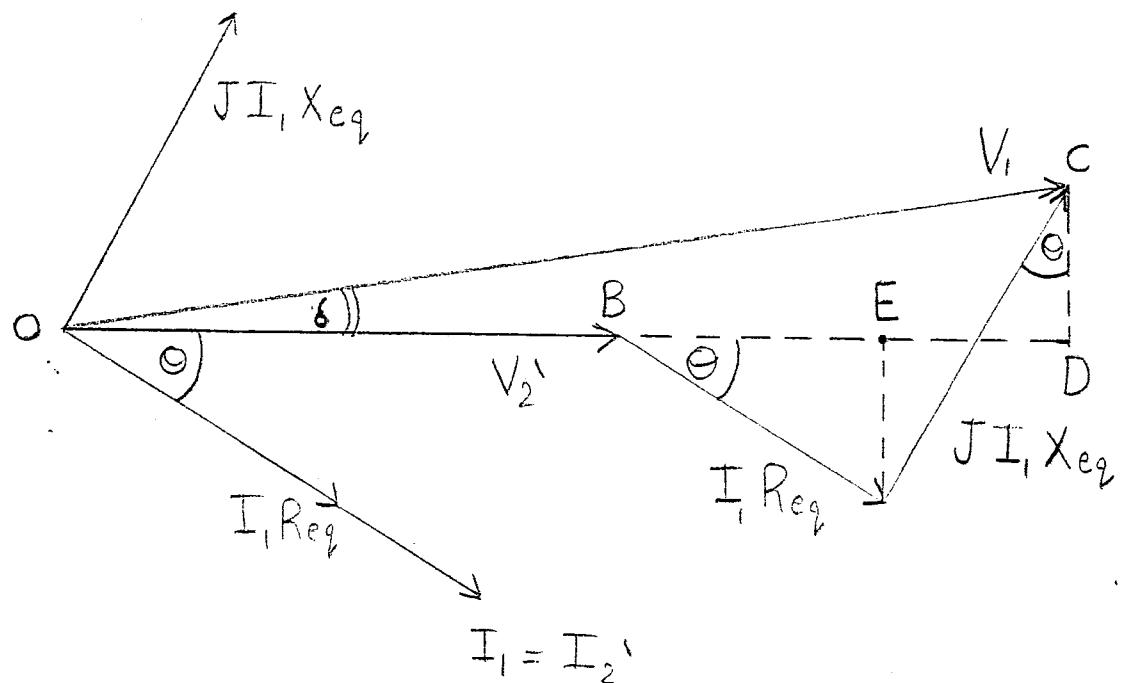


→ KVL

$$\overline{V_1} = \overline{V_2} + \overline{I_1} (R_{eq} + J X_{eq})$$

$$\overline{V_1} = \overline{V_2} + \overline{I_1} R_{eq} + J \overline{I_1} X_{eq} \rightarrow \text{phasor equation}$$

→ phasor diagram



To get $|V_1|$

$$|V_1| = OC \simeq OD \quad (\text{as } \delta \text{ is very small angle})$$

(3)

$$\begin{aligned}
 |V_1| &= \text{OD} = \underset{\downarrow}{\text{OB}} + \underset{\downarrow}{\text{BE}} + \text{ED} \\
 &= |V_2'| + I_1 R_{eq} \cos \theta + I_1 X_{eq} \sin \theta
 \end{aligned}$$

$$\therefore |V_1| = |V_2'| + I_1 R_{eq} \cos \theta + I_1 X_{eq} \sin \theta$$

$$|V_1| - |V_2'| = I_1 R_{eq} \cos \theta + I_1 X_{eq} \sin \theta$$

$$\therefore VR = \frac{|V_1| - |V_2'|}{|V_1|}$$

$$\therefore VR = \frac{I_1 R_{eq} \cos \theta + I_1 X_{eq} \sin \theta}{V_1} \quad \text{For lagging PF}$$

→ Condition for Maximum Voltage Regulation

$$\frac{d(VR)}{d\theta} = 0 \Rightarrow \frac{d}{d\theta} \left[\frac{I_1 R_{eq} \cos \theta + I_1 X_{eq} \sin \theta}{V_1} \right] = 0$$

$$\therefore I_1 R_{eq} (-\sin \theta) + I_1 X_{eq} \cos \theta = 0$$

$$\therefore \tan \theta = \frac{X_{eq}}{R_{eq}}$$

(4)

∴ The load angle for maximum VR is

$$\Theta_{\max} = \tan^{-1} \frac{X_{eq}}{R_{eq}} \quad (\text{lagging pf})$$

$$\therefore VR_{\max} = \frac{I_1 R_{eq} \cos \Theta_{\max} + I_1 X_{eq} \sin \Theta_{\max}}{V_1}$$

(5)

③ Efficiency of transformers (η)

$$\eta = \frac{P_o/p}{P_i/p} = \frac{P_o/p}{P_o/p + P_{loss}}$$

where $P_{loss} = P_{cu} + P_{core}$

\Rightarrow To get a general expression for η \rightarrow प्र०

$$\eta = \frac{P_o/p}{P_o/p + P_{cu} + P_{core}}$$

* Let X = loading ratio = loading factor

$$X = \frac{I_2}{I_{2F.L}}$$

$$X = \frac{I_2 * V_2}{I_{2F.L} * V_2} = \frac{S}{S_{F.L}}$$

$$\therefore X = \frac{I_2}{I_{2F.L}} = \frac{S}{S_{F.L}}$$

$$\Rightarrow ① P_o/p = V_2 I_2 \cos \phi_L = V_2 I_{2F.L} \left(\frac{I_2}{I_{2F.L}} \right) \cos \phi_L$$

$$P_o/p = X S_{F.L} \cos \phi_L \rightarrow ①$$

$$\Rightarrow ② P_{core} = \text{Constant} \rightarrow ②$$

$$\Rightarrow ③ P_{cu} \propto I^2$$

$$P_{cu} = X^2 P_{cu, F.L.} \rightarrow ③$$

From ①, ②, ③ we get:

$$\eta = \frac{X S_{F.L} \cos \phi_L}{X S_{F.L} \cos \phi_L + X^2 P_{cu, F.L} + P_{core}}$$

tie

To get η_{max}

$$\frac{d\eta}{dX} = 0 \Rightarrow X|_{\eta_{max}} = \sqrt{\frac{P_{core}}{P_{cu, F.L}}}$$

$$\eta_{max} = \frac{\left(\sqrt{\frac{P_{core}}{P_{cu, F.L}}} \right) * S_{F.L} * \cos \phi_L}{\left(\sqrt{\frac{P_{core}}{P_{cu, F.L}}} \right) * S_{F.L} * \cos \phi_L + \frac{1}{2} P_{core}}$$

tie

Note: Transformer efficiency is very high ($\eta_{trans} \approx 95\%$) as it has no rotational losses

Ex: ①

The parameters of the equivalent circuit of a 10kVA, 50Hz, 2300/230V distribution transformer are:

$$r_1 = 3.6 \Omega, x_1 = 15.8 \Omega$$

$$r_2 = 0.0396 \Omega, x_2 = 0.158 \Omega$$

Where 1, 2 refers to HV, LV respectively

- ① If the transformer delivers its rated KVA at 0.8 pf lagging to a load on low-tension (LV) side, calculate Voltage Regulation.
- ② Calculate the efficiency in part (a) if the iron losses are 75W at rated voltage.

⑧

Solution

① To get I_2 rated at LV side

$$|I_{2\text{rated}}| = \frac{S_{\text{rated}}}{V_2} = \frac{10000}{230} = 43.48 \text{ A}$$

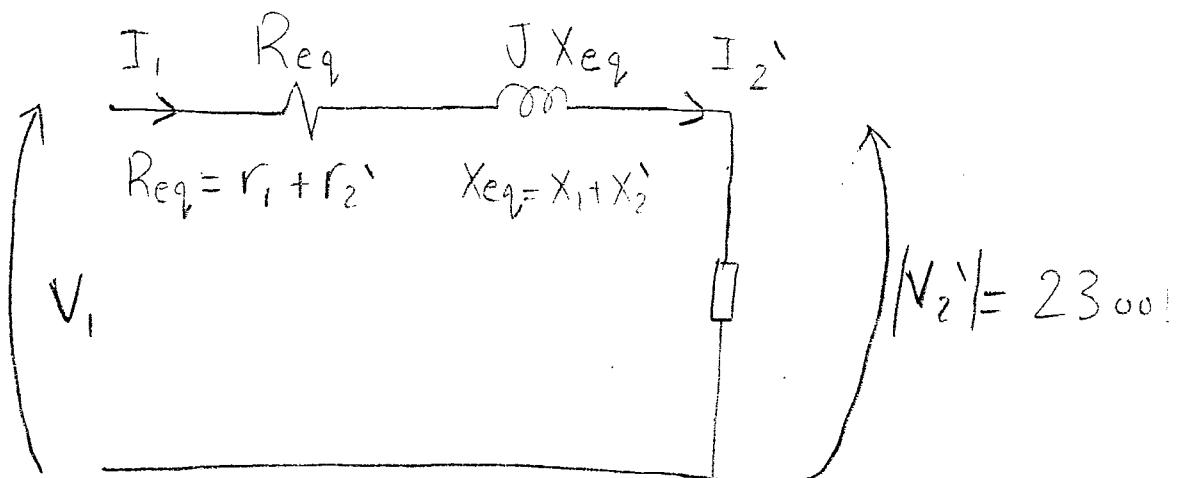
Referring to HV side

$$|V_2'| = a |V_2| = 10 \times 230 = 2300 \text{ V}$$

$$|I_2'| = \frac{1}{a} |I_2| = \frac{1}{10} \times 43.48 = 4.348 \text{ A}$$

$$r_2' = a^2 r_2 = (10)^2 \times 0.0396 = 3.96 \text{ N}$$

$$X_2' = a^2 X_2 = (10)^2 \times 0.158 = 15.8 \text{ N}$$



$$R_{\text{eq}} = r_1 + r_2' = 7.56 \text{ N}$$

$$X_{\text{eq}} = X_1 + X_2' = 31.6 \text{ N}$$

$$\Phi = \cos^{-1} 0.8 = 36.87^\circ \text{ (lagging)}$$

⑨

Using laws of Voltage regulation : ($I_1 = I_2$)

$$V_1 - V_2' = I_1 R_{eq} \cos \phi + I_1 X_{eq} \sin \phi$$

$$V_1 = V_2' + I_2' R_{eq} \cos \phi + I_2' X_{eq} \sin \phi$$

$$\therefore V_1 = 2300 + (4.348)(7.56) \cos(36.87) \\ + (4.348)(31.6) \sin(36.87)$$

$$V_1 = 2408.7 \text{ V}$$

$$\therefore VR = \frac{V_1 - V_2'}{V_1} = \frac{2408.7 - 2300}{2408.7}$$

$$VR = 4.5\%$$

(b) Efficiency

$$\eta = \frac{X S_{F,L} \cos \phi}{X S_{F,L} \cos \phi + X^2 P_{Cu,F,L} + P_{core}}$$

$$* X = 1$$

$$* S_{F,L} = 10000 \text{ VA} , \cos \phi = 0.8$$

$$* P_{Cu,F,L} = (I_2')^2 R_{eq} = (4.348)^2 * 7.56 = \underline{143 \text{ W}}$$

$$* P_{core} = P_{iron} = \underline{75 \text{ W}}$$

$$\therefore \eta = 97.34\%$$

(10)

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Given: $S_{F.L} = 500 \text{ KVA}$

$$\bullet \eta \Big|_{x=1} \text{ (Full load)} = 0.95$$

$$\bullet \eta \Big|_{x=0.6} = 0.95$$

$$\bullet \text{P.F} = 1$$

a) Separate out the transformer losses

Using the law of η

$$\eta = \frac{P_o/p}{P_i/p} = \frac{P_o/p}{P_o/p + P_{loss}} = \frac{P_o/p}{P_o/p + P_{Cu} + P_{core}}$$

$$\eta = \frac{X S_{F.L} \cos \phi_i}{X S_{F.L} \cos \phi_i + X^2 P_{Cu,F.L} + P_{core}}$$

$$\eta \Big|_{x=1} = \eta \Big|_{x=0.6} = 0.95$$

$$\Rightarrow \eta|_{x=1} = 0.95$$

$$\frac{1 \times 500 \times 1}{1 \times 500 \times 1 + (1)^2 P_{Cu_{F.L}} + P_{core}} = 0.95$$

$$1 + P_{Cu_{F.L}} + P_{core} = 26.31 \text{ kW}$$

$$\therefore P_{Cu_{F.L}} + P_{core} = 26.31 \text{ kW} \rightarrow ①$$

$$\Rightarrow \eta|_{x=0.6} = 0.95$$

$$\frac{0.6 \times 500 \times 1}{0.6 \times 500 \times 1 + (0.6)^2 \times P_{Cu_{F.L}} + P_{core}} = 0.95$$

$$0.36 P_{Cu_{F.L}} + P_{core} = 15.79 \text{ kW}$$

$$\therefore 0.36 P_{Cu_{F.L}} + P_{core} = 15.79 \text{ kW} \rightarrow ②$$

Solving ① & ② $\Rightarrow P_{Cu_{F.L}} = 16.4 \text{ kW}$
 $P_{core} = 9.87 \text{ kW}$

$$(b) \eta|_{x=0.75}, p.f = 1$$

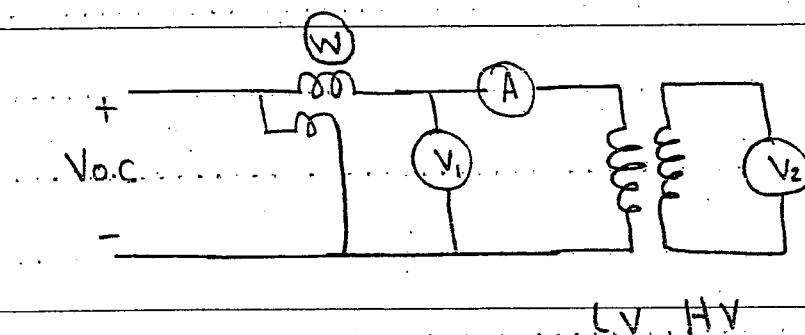
$$\eta|_{x=0.75} = \frac{0.75 \times 500 \times 1}{0.75 \times 500 \times 1 + (0.75)^2 \times 16.4 + 9.87}$$

$$\eta|_{x=0.75} = 95.15\% \quad \#$$

(IV). Transformer Tests:

(1). Open circuit Test:-

This test is performed to find $(R_c \& X_m)$



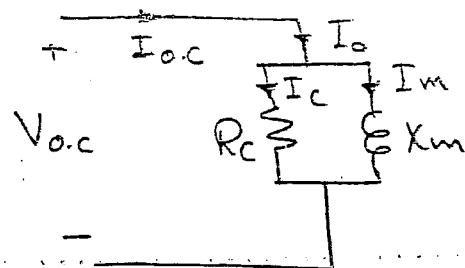
$\dots V_o.c \equiv$ Applied rated voltage $\equiv V_1$

$\dots I_o.c \equiv$ open circuit Current $\equiv A$

$\dots P_o.c \equiv$ open circuit transformer power $\equiv W$

How to get $R_c \& X_m$?

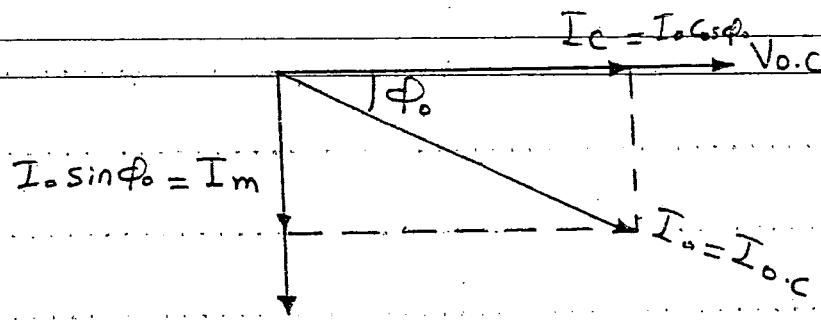
The equivalent circuit of the transformer during the open circuit test will be:



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$$\therefore R_C = \frac{V_{o.c}}{I_C} \quad , \quad X_M = \frac{V_{o.c}}{I_M}$$

The values of I_C & I_M can be calculated through:



$P_o.c$ will be used to get ϕ_o , then using the above phasor we draw $I_o.c$ lagging $V_o.c$ by ϕ_o , then resolve $I_o.c$ to $(I_C \& I_M)$.

$P_o.c = V_o.c I_o.c \cos\phi_o \Rightarrow \phi_o$ can be calculated.

$$\therefore \cos\phi_o = \frac{P_o.c}{(V_o.c)(I_o.c)} \Rightarrow \phi_o = 0.1$$

$$\therefore I_C = I_o \cos\phi_o \quad \& \quad I_M = I_o \sin\phi_o$$

$$\therefore R_C = \frac{V_{o.c}}{I_o.c} \quad \& \quad X_M = \frac{V_{o.c}}{I_M}$$

N.B:

1. $I_m = \sqrt{I_{o.c}^2 - I_c^2}$ (Another way).
2. R_c & X_m are referred to (L.V) side
3. $I_{o.c.}$ = no-load current
4. $\cos \phi_o$ = no-load power factor
5. $V_{o.c.}$ = rated voltage of the transformer if not given
6. Instruments are connected to L.V side (why?)
 usually $(I_{o.c.} = I_o)$ is very small compared to
 the full load current ($\approx 1.5 I_{\text{full load}}$).
 This small current is measured through the L.V.
 of the transformer for better accuracy.

N.B

- usually the current in H.V side is lower than that of L.V

$$\therefore V_1 I_1 = I_2 V_2 \Rightarrow \frac{I_2}{I_1} = \frac{V_1}{V_2}.$$

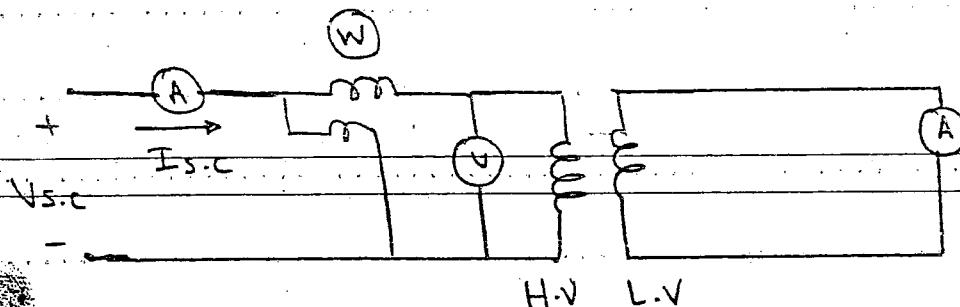
$$\rightarrow \text{If } \frac{V_1}{V_2} > 1$$

$$\therefore I_2 > I_1 \Rightarrow \text{our case.}$$

(15)

(2) Short Circuit test:-

- It is performed to find R_{eq} & X_{eq} .

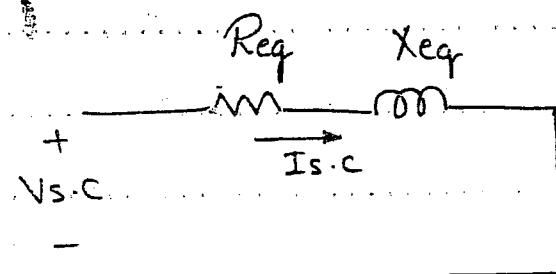


- $V_{s.c.} = 5-10\%$ of rated voltage of H.V side $\equiv \textcircled{V}$

- $I_{s.c.} = \text{short circuit transformer current} \equiv \textcircled{A}$

- $P_{s.c.} = \text{short circuit transformer power} \equiv \textcircled{W}$

- The equivalent circuit during this test:



- The $(R_s + X_s)$ branch is neglected as the current (I_o) is very small with respect to $I_{s.c.}$.

- To get (R_{eq}) :

$$R_{eq} = \frac{P_{s.c.}}{I_{s.c.}^2}$$

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To get Z_{eq} :

$$Z_{eq} = \sqrt{R_{eq}^2 + X_{eq}^2} = \frac{V_{s.c.}}{I_{s.c.}}$$

$$X_{eq} = \sqrt{\left(\frac{V_{s.c.}}{I_{s.c.}}\right)^2 - R_{eq}^2}$$

N.B.:

1. $I_{s.c.} \approx$ If.L. of transformer. H.V. side.

2. $(R_{eq} \& X_{eq})$ are referred to the H.V. side.

3. The instruments are placed on H.V. side. (why?)

(v_{ki} , a_{ki} , \bar{a}_{ki})

In this test, the Current flowing ($I_{s.c.}$) is...

very high. (\approx If.L.)

This current will be very high in the L.V. side and it will be very difficult to measure it by an ammeter.

The voltage ($V_{s.c.} \approx 5-10\% V_{rated}$), so it will be more accurate to measure it using the H.V side.

4. You can use:

$$r_1 = r_2' = \frac{R_{eq}}{2}$$

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$$x_1 = x_2' = \frac{x_{eq}}{2}$$

5. The open circuit test & short circuit test may be performed on any side, but it will be mentioned in the problem.

6. Default: (not mentioned in the problem).

Open circuit test \Rightarrow on L.V. side

Short circuit test \Rightarrow on H.V. side.

7. Cross-section Area of LV Winding

is large as it carries higher current

$\therefore R_{LV}$ is small ($R = \frac{PL}{A}$)

8. Cross-section Area of HV Winding (18)

is small

$\therefore R_{HV}$ is high

Ex ②

A 50 KVA, 2200/220 V transformer

When tested give the following results:

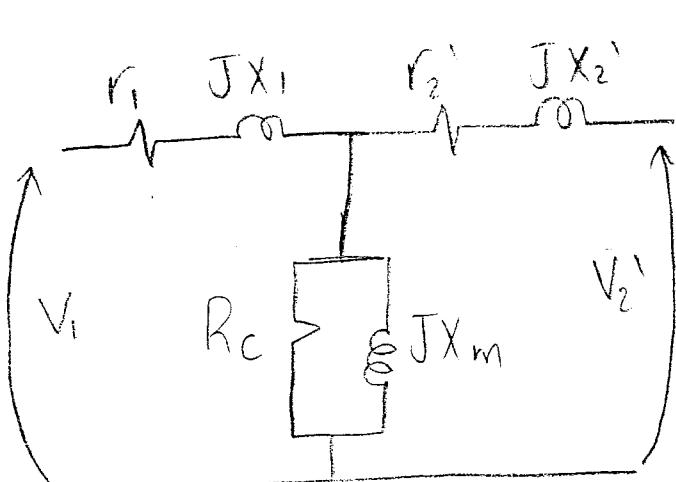
open circuit test: Measurements on LV side
40SW, SA, 220V

short circuit test: Measurements on HV side
80SW, 20.2 A, 95V

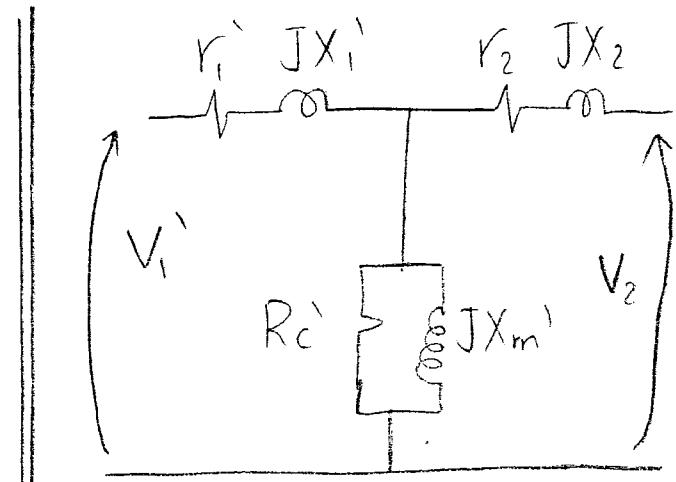
Find equivalent circuit parameters in
ohms referred to HV side & LV side

Solution

HV side \rightarrow primary side in this
problem



Exact equivalent circuit
Referred to HV side



Exact equivalent circuit
Referred to LV side

⑯

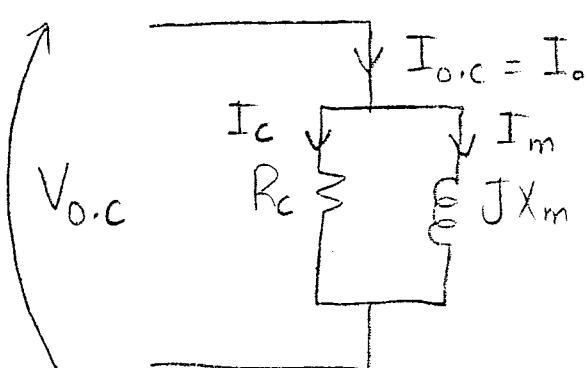
Solution

open circuit test

$$P_{o.c} = 40 \text{ SW}$$

$$I_{o.c} = 5 \text{ A}$$

$$V_{o.c} = 220 \text{ V}$$



$$\rightarrow * P_{o.c} = V_{o.c} I_{o.c} \cos \phi$$

$$\cos \phi = \frac{40}{220 \times 5} = 0.368$$

$$\boxed{\phi = 68.4^\circ}$$

$$\rightarrow I_c = I_{o.c} \cos \phi = 1.84$$

$$I_m = I_{o.c} \sin \phi = 4.65$$

$$\rightarrow R_c|_{LV} = \frac{V_{o.c}}{I_c} = 119.56 \Omega$$

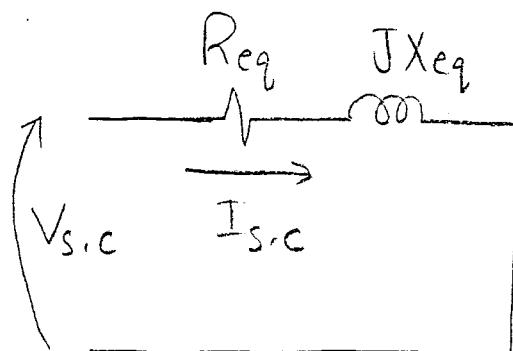
$$X_m|_{LV} = \frac{V_{o.c}}{I_m} = 47.3 \Omega$$

Short circuit test

$$P_{s.c} = 80 \text{ SW}$$

$$I_{s.c} = 20.2 \text{ A}$$

$$V_{s.c} = 95 \text{ V}$$



$$\rightarrow R_{eq} = \frac{P}{I_{s.c}^2}$$

$$R_{eq}|_{HV} = \frac{80}{(20.2)^2} = 1.97 \Omega$$

$$\rightarrow Z_{eq} = \frac{V_{s.c}}{I_{s.c}} = 4.7$$

$$\rightarrow X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$

$$X_{eq}|_{HV} = 4.26 \Omega$$

$$\therefore R_{eq}|_{HV} = 1.97 \Omega$$

$$X_{eq}|_{HV} = 4.26 \Omega$$

(20)

Exact equivalent circuit referred to

HV side

$$* R_C|_{HV} = R_C|_{LV} \left(\frac{N_{HV}}{N_{LV}} \right)^2 = 119.56 \left(\frac{2200}{220} \right)^2 = 11.95 \text{ kN}$$

$$* X_m|_{HV} = X_m|_{LV} \left(\frac{N_{HV}}{N_{LV}} \right)^2 = 47.3 \left(\frac{2200}{220} \right)^2 = 4.73 \text{ kN}$$

$$* R_{eq}|_{HV} = 1.97$$

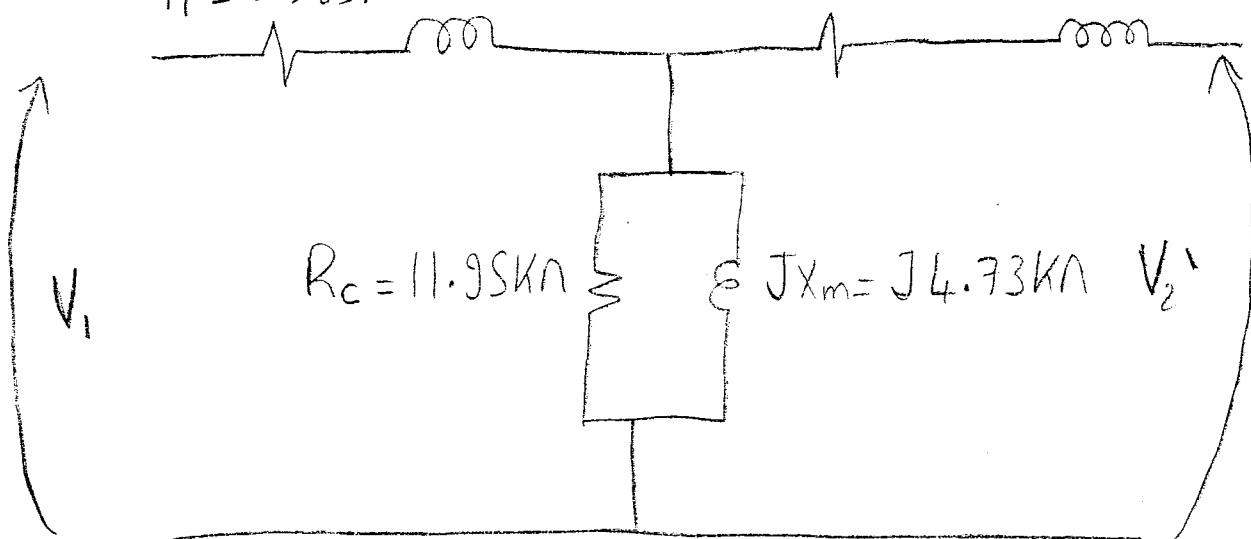
$$R_{eq}|_{HV} = r_1 + r_2' \quad (r_1 \approx r_2')$$

$$\therefore r_1 = r_2' = \frac{R_{eq}|_{HV}}{2} = 0.98 \text{ S} \Lambda$$

$$* X_{eq}|_{HV} = 4.26$$

$$\therefore X_1 = X_2' = \frac{4.26}{2} = 2.13$$

$$r_1 = 0.98 \text{ S} \Lambda \quad jX_1 = j2.13 \quad r_2' = 0.98 \text{ S} \Lambda \quad jX_2' = j2.13$$



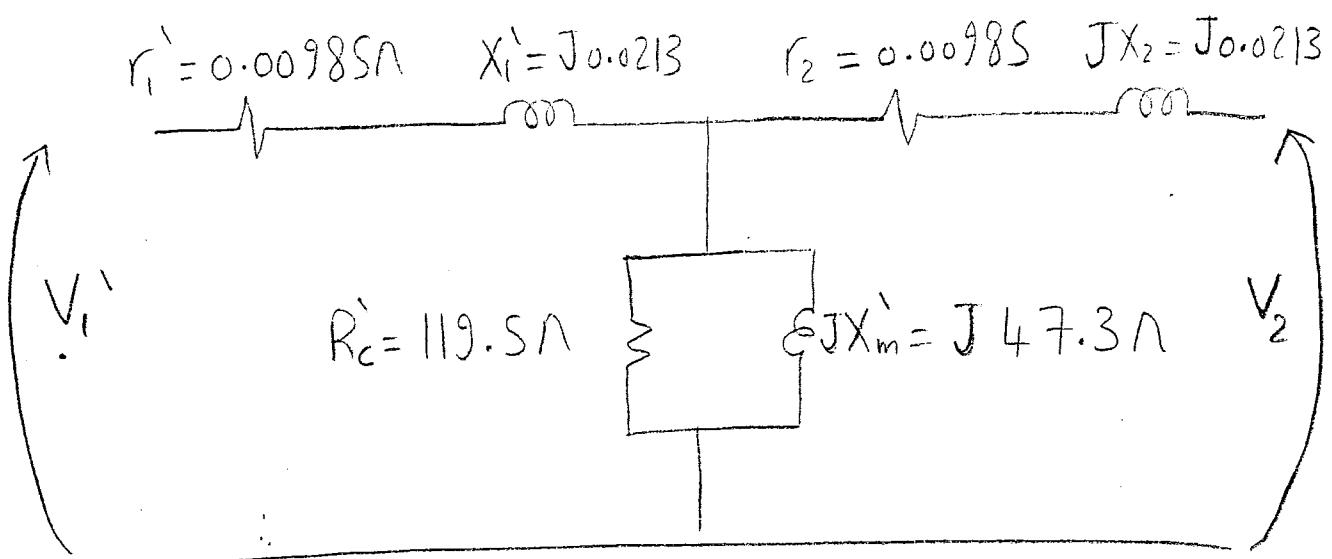
(21)

Exact equivalent circuit referred to
LV side

Referring the circuit of HV side
to LV side

$$r|_{LV} = r|_{HV} \left(\frac{N_{LV}}{N_{HV}} \right)^2 = r|_{HV} \left(\frac{220}{2200} \right)^2$$

$$r|_{LV} = \frac{r|_{HV}}{100}$$



Separation of iron losses

$$\rightarrow P_{\text{iron}} = P_{\text{core}} = P_h + P_e$$

→ where P_h : hysteresis losses

P_e : eddy losses

$$* P_h = K_h F (B_{\max})^x$$

x : Steinmetz Constant ($x=1.6$)

$$P_h = K_h F (B_{\max})^{1.6}$$

↳

$$* P_e = K_e F^2 (B_{\max})^2$$

↳

$$\text{Where } B_{\max} = \frac{V}{4.44 F N A}$$

$$\therefore P_h = K_h F \left(\frac{V}{4.44 F N A} \right)^x$$

$$P_e = K_e F^2 \left(\frac{V}{4.44 F N A} \right)^2$$

$$P_h = K_h' V^x F^{1-x}$$

$$P_e = K_e' V^2$$

Ex(3):

In a transformer, the core loss is found to be 52 W at 40 Hz and 90 W at 60 Hz measured at the same peak flux density. Compute the hysteresis and eddy current losses at 50 Hz

Solution

$$P_h = K_h F (B_{max})^{1.6}$$

$$\therefore B_{max} = \text{Constant}$$

$$P_h = K_h' F$$

$$P_e = K_e F^2 B_{max}^2$$

$$\therefore B_{max} = \text{Constant}$$

$$\therefore P_e = K_e' F^2$$

$$\therefore P_{core} = P_h + P_e$$

$$P_{core} = K_h' F + K_e' F^2$$

Using the given conditions

$$P_{core} = 52 \text{ W at } 40 \text{ Hz} \Rightarrow 52 = K_h' (40) + K_e' (40)^2 \quad (1)$$

$$P_{core} = 90 \text{ W at } 60 \text{ Hz} \Rightarrow 90 = K_h' (60) + K_e' (60)^2 \quad (2)$$

Solving ①, ② \Rightarrow $K_h = 0.9$ $K_e = 0.01$

\therefore at 50Hz

$$P_{core} = K_h (S_0) + K_e (S_0)^2$$

$$P_{core} = 0.9 (S_0) + 0.01 (S_0)^2$$

$$P_{core} = 4S + 2S$$

$$\therefore P_{core} = 70W$$

$$P_h = 4SW$$

$$P_e = 2SW$$